



# Optimal Control Design Results for Multiple Wheeled Mobile Robots

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## Abstract

This research presents the optimal control design results for the nonlinear trajectory tracking of multiple wheeled mobile robots (WMRs). The purpose of this design is to propose an optimal control method for the nonlinear trajectory tracking problem of multiple WMRs. The key contribution of this research is a simplified control structure that enables effective trajectory tracking for multiple WMRs. Generally, solving the nonlinear trajectory tracking problem for multiple WMRs is highly challenging. However, through a series of mathematical operations on the trajectory tracking error dynamics between the controlled WMRs and the desired trajectories, this trajectory tracking problem can be transformed into solving a nonlinear time-varying equation. Furthermore, the solution to this nonlinear time-varying equation can be derived in a simple form. Finally, to verify the performance of the proposed method, a testing scenario with an S-type reference trajectory is applied for performance evaluation.

**Keywords:** Wheeled Mobile Robot (WMR); Nonlinear Trajectory Tracking; Optimal Control

## Introduction

In recent decades, technological advances and the emergence of the digital era have led to the widespread use of multiple WMRs in daily life, most of which require a high-quality motion mechanism and control system. These robots are applied across various industrial and service sectors, including transportation, inspection, and security [1-4]. Consequently, precise motion control of multiple WMRs has garnered significant attention from researchers and the robotics industry [5-7]. A control structure that is straightforward to implement is essential for real-time tracking control in the design of multiple WMRs systems. As noted in existing research [8-16], developing an effective control design for multiple WMRs to precisely track predefined paths remains an open and challenging issue in robotics. The path-tracking system of multiple WMRs must achieve minimal error between the actual and desired paths, even under the influence of slippage, disturbances, and measurement noise. Therefore, accurately controlling multiple WMRs has become increasingly important.

## Simulation Configurations

To evaluate the trajectory tracking performance of multiple WMRs, simulations are conducted in MATLAB to track an S-type path using the proposed method.

The parameters of multiple WMRs used in this simulation validation are as follows: the weight of the wheeled mobile robot is  $m = 6.6$  (kg), the wheel radius is  $r = 2.8$  (cm), the width of the robot is  $w = 17.8$  (cm), and the distance from the center point to the wheels is  $d = 6$  (cm). Based on these values, we build a simulation environment that closely resembles the practical setup. A predefined S-type reference trajectory, generated by the equations shown below, is selected as the target trajectory for the simulation test.

Equation of a S-type reference trajectory:

$$x = x_0 + r_d \cos(2\theta_d)$$

$$y = y_0 + 2r_d \sin(\theta_d)$$

where  $r_d$  and  $\theta_d = \int w dt$  are the radius of the desired S-type reference trajectory and desired rotation angle with a constant predefined angular velocity  $w_p$ , respectively. The initial condition

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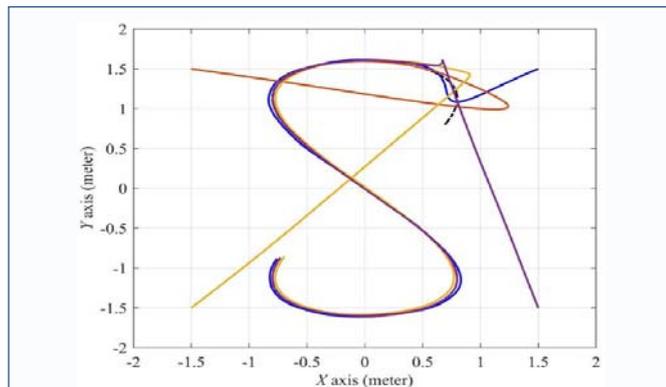
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of the desired trajectory is  $(x_0=0$  meter,  $y_0=0$  meter,  $r_d=0.8$  meter and  $w_d=3^0/s$ ), and the initializations of multiple WMRs using the proposed control method are:  $(x_c=1.5$  meter,  $y_c=1.5$  meter),  $(x_c=-1.5$  meter,  $y_c=1.5$  meter),  $(x_c=-1.5$  meter,  $y_c=-1.5$  meter), and  $(x_c=1.5$  meter,  $y_c=-1.5$  meter).

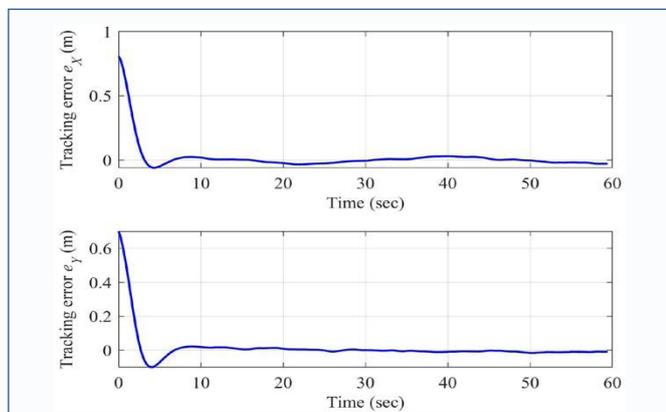
### Simulation Results

To verify tracking performance under varying initial conditions, four different starting positions are tested:

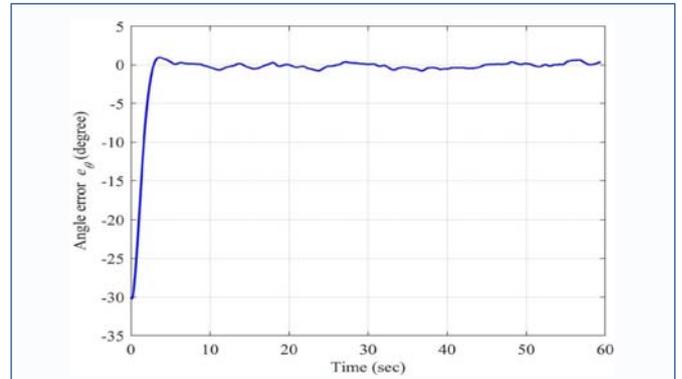
$(x_c=1.5$  meter,  $y_c=1.5$  meter),  $(x_c=-1.5$  meter,  $y_c=1.5$  meter),  $(x_c=-1.5$  meter,  $y_c=-1.5$  meter), and  $(x_c=1.5$  meter,  $y_c=-1.5$  meter). These conditions are chosen to evaluate the performance of multiple controlled robots under 30% external disturbances. The trajectory tracking history is shown in Figure 1, which illustrates the tracking paths on the X-Y plane in relation to a desired S-type trajectory under the proposed control design. As shown in Figure 1, the results indicate that the proposed design achieves a rapid convergence to the reference S-type trajectory from each of the four initial positions. Figures 2 to 9 display the tracking error histories for position and orientation relative to the four different initial points. These simulation results demonstrate the robustness of the proposed design's performance, even when multiple robots encounter significant unknown external disturbances.



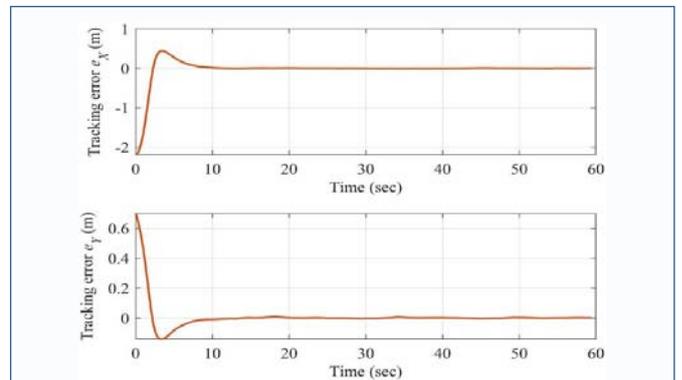
**Figure 1:** Trajectory Tracking Results of the Proposed Control Design for Four Different Initial Conditions  $(x_c=1.5$  meter,  $y_c=1.5$  meter),  $(x_c=-1.5$  meter,  $y_c=1.5$  meter),  $(x_c=-1.5$  meter,  $y_c=-1.5$  meter), and  $(x_c=1.5$  meter,  $y_c=-1.5$  meter).



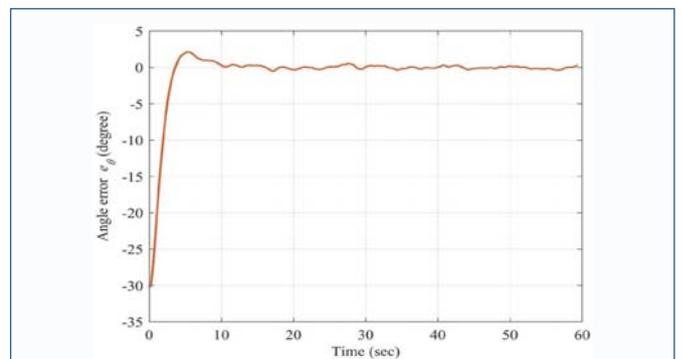
**Figure 2:** Tracking error results: X and Y axes relative to the initial point  $(x_c=1.5$  meter,  $y_c=1.5$  meter).



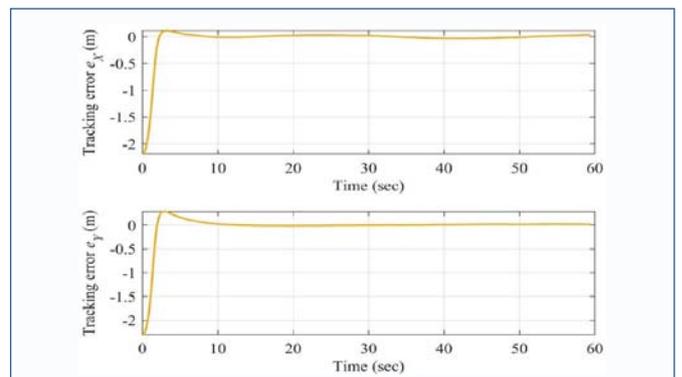
**Figure 3:** Tracking error result: Angle relative to the initial point  $(x_c=1.5$  meter,  $y_c=1.5$  meter).



**Figure 4:** Tracking error results: X and Y axes relative to the initial point  $(x_c=-1.5$  meter,  $y_c=1.5$  meter).



**Figure 5:** Tracking error result: Angle relative to the initial point  $(x_c=-1.5$  meter,  $y_c=1.5$  meter).



**Figure 6:** Tracking error results: X and Y axes relative to the initial point  $(x_c=-1.5$  meter,  $y_c=-1.5$  meter).

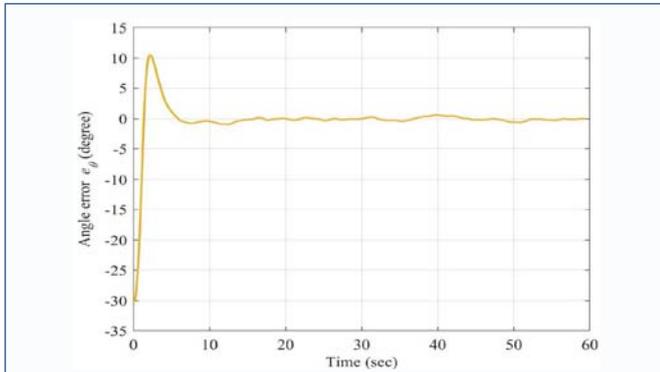


Figure 7: Tracking error result: Angle relative to the initial point ( $x_c=1.5$  meter,  $y_c=-1.5$  meter).

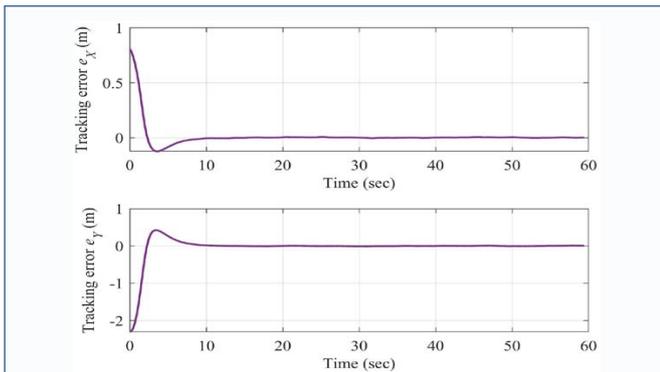


Figure 8: Tracking error results: X and Y axes relative to the initial point ( $x_c=1.5$  meter,  $y_c=-1.5$  meter).

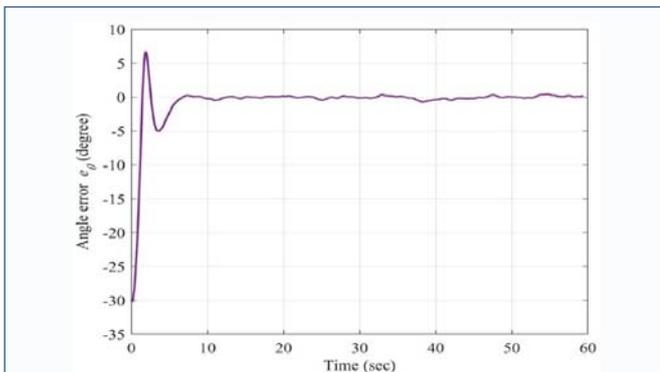


Figure 9: Tracking error result: Angle relative to the initial point ( $x_c=1.5$  meter,  $y_c=-1.5$  meter).

## Conclusion

In this research, the optimal control results for the trajectory tracking of multiple WMRs are presented. The proposed control method reduces design costs and computational load through the proposed solution, as demonstrated in this research. Generally, finding the proposed solution for this path-tracking problem is challenging. However, by applying an appropriate mathematical

transformation technique, an analytical solution for the nonlinear path-tracking problem in multiple WMRs can be obtained directly. Finally, the proposed method achieves satisfactory results in tracking the desired trajectory, as verified through simulations.

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